TZDII: Appendix B

When z is small ($|z| \ll 1$), one obtains a good approximation using just the first one or two terms of these five series. In particular, with $|z| \ll 1$, the binomial series reduces to the binomial approximation:

$$(1 + z)^n \approx 1 + nz \qquad \textbf{(binomial approximation)}$$

SOME INTEGRALS

Integrals of the form

$$I_n = \int_0^\infty x^n e^{-\lambda x^2}\, dx$$

where λ is a positive number, occur frequently in several branches of physics. When n is a positive integer, their value can be found from the following:

$$I_0 = \sqrt{\frac{\pi}{4\lambda}}, \quad I_1 = \frac{1}{2\lambda}, \quad I_2 = \sqrt{\frac{\pi}{16\lambda^3}}, \quad I_3 = \frac{1}{2\lambda^2}, \quad I_4 = \frac{3}{8}\sqrt{\frac{\pi}{\lambda^5}}$$

and

$$I_n = -\frac{dI_{n-2}}{d\lambda}$$

Notice that the integral $\int_{-\infty}^{\infty} x^n e^{-\lambda x^2}\, dx$ equals $2I_n$ when n is even, but is zero if n is odd.

Another common integral is the indefinite integral

$$J_n = \int x^n e^{-x/b}\, dx$$

When n is a small integer, this is easily evaluated by parts, for example,

$$J_0 = -be^{-x/b}, \qquad J_1 = -(b^2 + bx)e^{-x/b}, \qquad J_2 = -(2b^3 + 2b^2x + bx^2)e^{-x/b}$$

In general,

$$J_{n+1} = b^2 \frac{\partial J_n}{\partial b}$$

Note, in particular, that

$$\int_0^\infty e^{-x/b}\, dx = b, \qquad \int_0^\infty xe^{-x/b}\, dx = b^2, \qquad \int_0^\infty x^2 e^{-x/b}\, dx = 2b^3$$

and also that

$$\int_0^\infty \sqrt{x}e^{-x/b}\, dx = \frac{\sqrt{\pi b^3}}{2}$$

Plus the generalized form of the penultimate row of integrals

$$\int_0^\infty x^n e^{-(x/b)} dx = n!b^{(n-1)}$$